MODULATION OF NON-LINEAR WAVE IN A FLUID-FILLED THICK ELASTIC TUBE

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Every challenging work needs self efforts as well as guidance of elders, especially those who are very close to my heart.

My humble effort I dedicate to my sweet and loving

Father and Mother,

whose affection, love, encouragement, and prayers of day and night make me able to get such success and honour.

Along with the hard working and respected supervisor

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ABSTRACT

This research presents an analytical study on the wave modulation flow in an artery. The artery is simulated as an incompressible, isotropic, and thick walled elastic tube. By considering blood as an incompressible inviscid fluid or incompressible viscous fluid, two mathematical models of non-linear wave modulation in a thick elastic tube were developed. The modulation of the non-linear wave in the long wave approximation was investigated using the reductive perturbation method. The governing equation for the incompressible inviscid fluid model was shown to be the non-linear Schrodinger equation (NLSE). As the dissipative non-linear Schrodinger equation (DNLSE), the control equation of the incompressible viscous fluid model was derived. These governing equations have been sought progressive wave-type solutions. It is observed that solitary wave type solutions with variable amplitude are admitted by these two equations. The effects on the blood flow characteristics have been extracted graphically by radial displacement, radial speed, axial speed, tube pressure, and hydrostatic pressure. Graphical analysis on wave amplitude variation, wave width, and wave travel was performed to illustrate the clarification of these two models towards wave modulation. Results showed that wave propagated smoothly for the first model, while the second model displayed the wave propagated with decreasing of wave amplitude. It was found that as the fluid viscosity increased, the resistance for blood to flow also increased.

ABSTRAK

Kajian ini membentangkan kajian analitik mengenai tingkah laku aliran darah di dalam arteri. Arteri dimodelkan sebagai tiub anjal yang tidak dapat dimampat, isotropik dan berdinding tebal. Dengan mengambil kira darah sebagai bendalir tidak likat yang tidak dapat dimampatkan dan bendalir likat yang tidak dapat dimampatkan, dua modulasi gelombang tak linear dalam tiub elastik tebal akan terbentuk. Dengan menggunakan kaedah usikan penurunan, modulasi gelombang tak linear dalam penghampiran gelombang panjang dikaji. Persamaan yang menakluk model bendalir tidak likat yang tidak dapat dimampatkan ditunjukkan sebagai persamaan Schrodinger tak linear (NLSE). Persamaan kawalan untuk model bendalir likat yang tidak dapat dimampatkan diperolehi sebagai persamaan tak linear disipatif Schrodinger (DNLSE). Penyelesaian jenis gelombang progresif kepada persamaan penakluk ini dicari. Didapati bahawa kedua-dua persamaan ini memenuhi penyelesaian jenis gelombang bersendirian dengan amplitud yang berubah-ubah. Kesan anjakan radial, halaju radial, halaju paksi, tekanan tiub dan tekanan hidrostatik pada ciri-ciri aliran darah diperoleh dan ditunjukkan secara grafik. Analisis grafik mengenai variasi amplitud gelombang, lebar gelombang dan perjalanan gelombang dilakukan untuk menggambarkan klarifikasi kedua-dua model ke arah modulasi gelombang. Hasilnya menunjukkan gelombang bergerak dengan lancar untuk model yang pertama, sementara itu model yang kedua menunjukkan gelombang bergerak bersama penurunan amplitud gelombang. Ini didapati, apabila kelikatan darah meningkat maka halangan untuk darah mengalir juga meningkat.

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LIST OF SYMBOLS AND ABBREVIATIONS

A	-	inner cross-sectional area
a_r^*	-	radial acceleration component
B_i	-	ratio of the deformed inner cross-sectional
C	-	unknown function
$c.c$	-	complex conjugate of the corresponding expressions
c_{kl}	-	Finger deformation tensor
c	-	scale parameter
c_0	-	speed of the wave
D	-	integration constant
H	-	initial of tube thickness
h	-	thickness of the tube
I_1	-	first invariant of Finger deformation tensor
I_2	-	second invariant of Finger deformation tensor
I_3	-	third invariant of Finger deformation tensor
i	-	imaginary number
K	-	coefficient of elastic
k	-	wave number
l	-	length
m	-	inertia parameter
P	-	pressure of hydrostatic
P^*	-	dimensional fluid pressure in the inner surface of the tube
P_r^*	-	dimensional fluid reaction forc in the radial direction
$\overline{P}$	-	dimensional pressure function of fluid
P_r	-	fluid reaction force
P_1 & P_2	-	pressure at two different points

p	-	nondimensional fluid medium pressure
p^*	-	dimensional fluid medium pressure
$\overline{p}$	-	tube pressure
Q	-	axial velocity
R	-	flow resistance
R_i	-	inner radius in the undeformed configuration of the tube
R_0	-	radius of circular conical tube
r_i	-	inner radii of the tube after finite static deformation
r_0	-	outer radii of the tube after finite static deformation
r^4	-	radius to the fourth power
t	-	non-dimensional time parameter
t_{kl}	-	Cauchy stress tensor for the final configuration
t^*	-	dimensional time parameter
U	-	radial displacement of artery
U^*	-	complex conjugate of U
u	-	wave
V	-	radial velocity
V_z^*	-	components of fluid velocity for axial directions
V_r^*	-	components of fluid velocity for radial directions
$\overline{V}$	-	kinematic viscosity
v	-	non-dimensional radial fluid velocity
v^*	-	axial velocity of the fluid
v_0	-	constant
w^*	-	component of fluid velocity
z	-	axial parameter
z^*	-	dimensional axial parameter
α & β	-	constant
δ_{kl}	-	Kronecker delta
ε	-	small parameter for nonlinearity and dispersion
η	-	coefficient for viscous

λ	-	constant
λ_z	-	axial stretch ratio
λ_θ	-	circumferential stretch ratio
μ	-	shear modulus for tube material
μ_1	-	coeficient of the dispersive term
μ_2	-	coeficient of the nonlinear term
μ_3	-	coeficient of the dissipative term
μ_v	-	viscosity of fluid
υ	-	viscosity
$\bar{\upsilon}$	-	non-dimensional viscosity ($\bar{\upsilon} = \varepsilon^2 \upsilon$ in chapter 5)
ω	-	angular frequency
ρ_f	-	mass density
Σ	-	density function of strain energy
τ	-	time variable
ξ	-	space variable
ζ	-	constant
ϕ	-	distorted tapering angle
Φ	-	tapering angle
IST	-	inverse scattering transform
KdV	-	Korteweg-de Vries
KdVB	-	Korteweg-de Vries-Burger's
KP	-	Kadomtsev Petviashvili
NLS	-	nonlinear Schrodinger
ODE	-	ordinary differential equation
PDE	-	partial differential equation
PLK	-	Poincare-Lighthill-Kuo
RPM	-	reductive perturbation method
SITP	-	short in time pulse
SISP	-	short in space pulse

LIST OF APPENDICES

LIST OF PUBLICATION

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CHAPTER 1

INTRODUCTION

1.1 General introduction

Many scientists have researched the artery that is part of the human circulatory system (also known as the cardiovascular system or vascular system). This system is used to transport nutrients, platelets, oxygen, carbon dioxide, and hormones in the body.

More than 400 years ago, William Harvey (1578-1657) discovered the history of the blood circulation (Ribatti, 2009). William Harvey was an English physician who first knew how to completely portray the deliberate course and properties of blood being pumped by the heart to the brain and body. In his exploration, Harvey concentrated more on the mechanics of blood-stream in the human body. By observing the activism of the heart in living animal, Harvey was able to see that systole was the dynamic period of the heart's development, pumping out the blood by its solid withdrawal.

Ribatti emphasized Harvey's research on observing the notion of heart in living animals in 2009. Unfortunately, in Harvey's research, he could not see vessels connecting an artery to the veins. At long last, Harvey suggested the presence of small fine anastomoses amongst supply routes and veins, but Marcello Malpighi did not find these until 1661. By using one of the early-invented microscope, Malpighi discovered vessels that connect artery to the veins (Pearce, 2007).

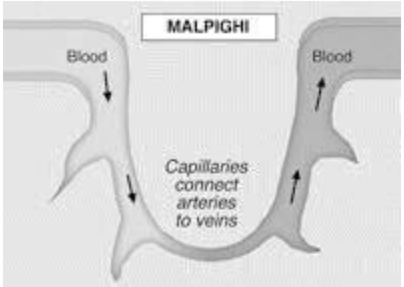

Figure 1.1: Malpighi discovered a vessel connecting an artery to the veins (Hwa & Aird, 2007)

Based on Figure 1.1, Hwa and Aird (2007) illustrated the circulation model based on Malpighi's research. The capillaries had been observed by using a microscope with a double-convex lens. Their discoveries show that walls channels and the walled tube are connected.

The arterial wall involves blood solutes transport phenomena (Diller *et al.*, 1980). The key segments of hemodynamics demonstrate the scientific conditions of liquid elements. Blood is a suspension of particles in a liquid called plasma consists of water (90-92%), proteins (7%), and inorganic components. Velocity and pressure are the principal quantities that describe blood flow in arteries (Quarteroni *et al.*, 2001). Removal of weight, vessel speed, and divider will be time capacity and spatial position. A feature of blood flows in the human body is represented by its pulsatility (Oscuii *et al.*, 2007). The blood flows in time with some approximation. The size of arteries has its own effect on shear stress, for example, the rate of shear stresses will be very low and the blood in arteries is treated as a non-Newtonian fluid (Nicholas and O'Rourke, 2005).

Blood is said to be Newtonian if it follows Newton's viscosity law, which is the shear stress is proportional to the rate of shear and its viscosity is the proportionality constant (Bessonov *et al.*, 2016). The flow of rate Q is from high pressure to low pressure. The flow rate will be higher if the pressure difference between two points is greater. It can be stated as:

$$Q = \frac{P_2 - P_1}{R}, \tag{1.1}$$

where P_1 and P_2 are pressures at two different points, while R stands for the flow resistance. The resistance R is given by

$$R = \frac{8\eta l}{\pi r^4}, \tag{1.2}$$

where equation (1.2) is called the Poiseuille's Law for resistance.

Demiray (1999) studied on the propagation of weakly non-liner waves in a fluid-filled thick viscoelastic tube. He used reductive disturbance method to investigate the propagation of a weakly non-linear wave in the long-wave approximation. He demonstrated by a legitimate scaling that the general equations were reduced to Korteweg-de Vries equation, Burgers' equation, Korteweg-de Vries-Burgers' (KdVB) equation, and the generalized Burgers' equation.

Demiray (2001) investigated on the modulation of non-linear waves in a viscous fluid contained in an elastic tube. For this model, he used the reductive perturbation technique and then obtained for this model the dissipative non-linear Schrodinger equation. Subsequently, Demiray (2005) carried out research on the head-on collision of solitary waves in a fluid-filled elastic tube. In this research, he used the extended Poincare-Lighthill-Kuo (PLK) perturbation method. The result of this study showed that the head-on collision of solitary waves was elastic. Demiray (2008) conducted a research on the modulation of non-linear waves in a fluid-filled thin elastic tube with a stenosis. By utilizing the reductive perturbation technique, the non-liner Schrodinger (NLS) equation was acquired.

1.2 Background of the problem

The blood wall material is known to be incompressible, anisotropic, and viscoelastic (Demiray, 2004). However, the arterial wall material is assumed to be incompressible, homogeneous, isotropic, and elastic due to its mathematical simplicity in mathematical modeling (Lambossy, 1951).

In studying the shock waves in the aorta mathematical model, Rudinger (1970) investigated the propagation of finite-amplitude waves in fluid-filled elastic or viscoelastic tubes by using the characteristics method. In 1997, Demiray investigated the effect of initial twisting on wave characteristics in an elastic thin tube filled with a prestressed fluid. It was difficult to see the harmonic wave type of solution for the field equations and to use the boundary conditions (Demiray, 1997).

Demiray (1998) used the reductive perturbation method to investigate non-linear wave modulation in a fluid-filled thick elastic tube. It has been shown that a non-linear Schrodinger (NLS) equation represents the adequacy balance of these waves.

1.3 Problem statement

In recent year, there is growing interest in studying the non-linear wave modulation in blood flow in the artery. Mostly, the research primarily focused on the propagation of waves in the arteries (see in Section 2.5). Most of the studies were concerned on wave modulation in the thin elastic tube. The studies of non-linear wave modulation in a fluid-filled thick elastic tube are rarely found in the literature. There is much interest in considering the blood as approximate equations of fluid. Studies on wave modulation with exact fluid equations are limited because the mathematical model is difficult to perform (Ravindran & Prasad, 1979).

The work in this study was motivated by considering the artery to be made of thick elastic tube, and two types of fluids, which were inviscid fluid and viscous fluid, were considered for the blood flows in the artery. The modulation of non-linear wave in the fluid-filled thick elastic tube would be observed by using the method of reductive perturbation. At the end, two different mathematical models for wave modulation in thick elastic tube were developed.

1.4 Objectives of the research

Objectives of this study are to:

i. derive the non-linear evolution equation for wave modulation in a thick elastic tube filled with inviscid fluid by using the reductive perturbation method.
ii. derive the non-linear evolution equation for wave modulation in a thick elastic tube filled with viscous fluid by using the reductive perturbation method.
iii. solve the non-linear evolution equations (i) and (ii) analytically.

iv. analyze the graphical output for the solution of non-linear evolution equations (i) and (ii).

1.5 Scope of the study

The human artery is made of a thick viscoelastic tube (Nicholas and O'Rourke, 2005). For this research, the mathematical model for modulation of non-linear wave in the fluid-filled thick elastic tube was derived by using the method of reductive perturbation. The artery was considered as a thick elastic tube whereas blood was assumed to be an incompressible inviscid fluid and incompressible viscous fluid. By using the reductive perturbation technique, non-linear evolution equations were obtained. Then, using a progressive wave solution, the non-linear evolution equation was solved analytically.

1.6 Significance of the study

The circulatory system plays a role as a transport system that transports nutrients, oxygen, carbon dioxide, and blood cells to the human body and it allows blood to circulate throughout the human body. The expectation of this research is to contribute the mathematical model for modulation of non-linear waves in a fluid-filled thick elastic tube. The method of progressive wave solution will be implemented to find the analytical solution for the corresponding evolution of the non-linear equation.

Then, MATLAB programming software would be used to obtain the graphical output. Through this research, mathematicians will be interested to know more about the evolution of non-linear equation and more experts will be created in the field of non-linear waves such as the medical diagnosis data at Universiti Tun Hussein On Malaysia (UTHM) and Malaysia.

1.7 Outline of research

This research focuses on the mathematical model for the blood flow in thick elastic tube full-filled with two different types of fluid that are inviscid fluid and viscous fluid. This section is about the main content of the research and it serves as an outline for quick reference to the appropriate section. This thesis is divided into seven chapters.

In Chapter 1, it provides an overview of what will be discussed in the following chapter. Chapter 1 includes a general introduction, the research background, the problem statement, scopes of study, and the significance of study. The outlines of this research are given at the end of this chapter.

For Chapter 2, starting with the history of solitary waves and Korteweg-de Vries (KdV) equation, the general form of the KdV equation and the applications of the KdV equation will be stated in this section. The history of non-linear Schrodinger (NLS) equation and its analytical solution will be explained in brief in the following sub-chapter. Then, the research carried out by previous researchers in Section 2.4 on the modulation of waves and propagation in an elastic tube with different fluid types in Section 2.5, there will be a simple brief on the purpose of the thick elastic tube used in this research and the equation of thick elastic tube. At last, in the last section of Chapter 2, past research on the reduction and perturbation method will then be discussed

In Chapter 3, the methodology of research to solve wave modulation in an inviscid fluid contained in a thick elastic tube will be discussed. The equations of inviscid fluid will be shown in the following sub-chapters. Moreover, non-dimensional quantities are then added to obtain the dimensionless equations of tube and fluids with its boundary conditions. In order to obtain the various order of differential equations, the technique of reductive perturbation will be implemented. Then, the non-linear Schrodinger equation will be obtained by solving a set of various order of differential equation.

Then, in Chapter 4, by using the progressive wave solution, the non-linear evolution equation obtained from the previous chapter will be solved analytically. Using MATLAB software, the graphical output for this model is plotted and the result from this section will be discussed.

The next chapter (Chapter 5) will discuss on the research methodology to solve the second model. The following sub-chapter shows the equation of viscous fluid. Next, by introducing the dimensional quantities, the dimensionless equation of fluids and tube with boundary conditions will be obtained. The various order of differential equations will be obtained by applying the reductive perturbation method. Further, the various order of differential equation will be solved to obtain the dissipative non-linear Schrodinger equation.

The dissipative non-linear Schrodinger equation will be solved analytically in Chapter 6 by using the progressive wave solution and the graphical output acquired by using MATLAB software. The result obtained from the graphical output will be discussed respectively.

In Chapter 7, which is part of the conclusion, the overall discussion on Chapter 3 - Chapter 6 will be concluded, including the part of interpretations and findings. The recommendations for further research will be explained in the last section of this chapter. References quoted are listed after Chapter 7 in the reference section.

CHAPTER 2

LITERATURE REVIEW

2.1 The discovery of solitary waves

At the beginning of 1834, a young Scottish architect named John Scott Russell (1808-1882) conducted tests to decide on the most effective outline for a channel ship. Based on Russell's observation, it demonstrated that soliton was the self-fortifying of a singular wave and shape that was kept up while proliferating at a consistent speed. Solitons are formed when non-linear and scattered effects are cancelled (Drazin and Johnson, 1989). This is a weird experiment that had been done by Russel because he could not prove the conclusion that he made before and tried to make physicians to believe it at that time. Since then, physicians had an argument among them over solitary waves until Diederik Korteweg and Gustav de Vries derived the Korteweg-de Vries equation, which modeled the solitary waves that Russel had observed.

There are some properties of solitary waves that have been found. First and foremost, the shape of a solitary wave is hyperbolic secant. If the initial mass of water is large, the solitary wave will be separated into two or more that separate in time. So, the separated solitary waves can interact each other easily without change of any kind. Lastly, the solitary waves are different from the normal waves because they will never merge and a larger one overtakes the smaller amplitude wave.

The Korteweg-de Vries (KdV) equation is a numerical model of a wave on shallow water surfaces. The KdV equation was first presented by Boussinesq in 1877 and it was rediscovered by Diederik Korteweg and Gustav de Vries in 1895 on the basis of solitary wave and periodic cnoidal wave solutions. In a computational study, Zabusky and Martin Kruskal demonstrated soliton behavior in a media subject to the

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